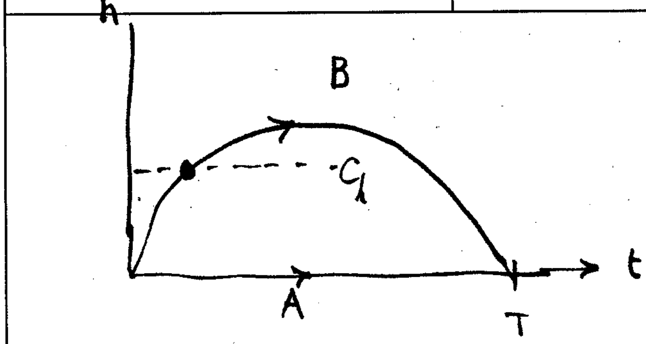


C_h is at a fixed height h coincides with location of B at some time.

$$d\tau_B = dt_c \sqrt{1 - \left(\frac{dh}{dt_c}\right)^2 \frac{1}{c^2}} \qquad d\tau_A = dt_A$$

Need to convert time measured by c to time measured by A.

$\omega_c \left(1 + \frac{gh}{c^2}\right) = \omega_A$ for a photon falling from c to A.

$$\Rightarrow \frac{dt_c}{1 + gh/c^2} = dt_A$$

Excess time $= \tau_B - \tau_A$

$$\Delta\tau = \int_0^T dt \cdot \left\{ \left(1 + \frac{gh}{c^2}\right) \sqrt{1 - \left(\frac{dh}{dt}\right)^2 \frac{1}{c^2(1 + gh/c^2)^2}} - 1 \right\}$$

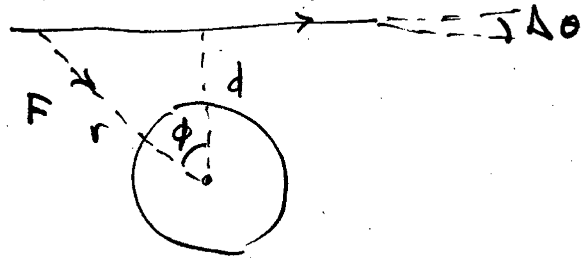
[Note: The total time $\tau_A = T$ is held fixed.]

$$= \int_0^T dt \left\{ \sqrt{\left(1 + \frac{gh}{c^2}\right)^2 - \frac{1}{c^2}\left(\frac{dh}{dt}\right)^2} - 1 \right\}$$

Euler-Lagrange Equation:

$$\frac{d}{dt}\left\{ \frac{-\dot{h}/c^2}{\left[\left(1 + \frac{gh}{c^2}\right)^2 - \dot{h}^2\right]^{1/2}} \right\} = \frac{g/c^2}{\left[\left(1 + \frac{gh}{c^2}\right)^2 - \dot{h}^2\right]^{1/2}}$$

In Non-relativistic limit, reduces to $\ddot{h} = -g$



$\Delta\theta$ = angle of deflection

$$= \frac{\Delta p}{p}$$

$$\Delta p = \int F_\perp \, dt = \int \frac{GM\left(\frac{\hbar\omega}{c^2}\right)}{r^2} \cdot \cos\phi \, dt$$

$$L = \text{angular momentum} = \left(\frac{\hbar\omega}{c^2}\right) . c . d = \frac{\hbar\omega}{c} d$$

$$= m r^2 \dot\phi = \left(\frac{\hbar\omega}{c^2}\right) r^2 \dot\phi$$

$$\Rightarrow \quad dt = d\phi \; \frac{m r^2}{m c d} = \frac{r^2}{cd} \, d\phi$$

$$\therefore \quad \Delta p = \frac{GM\hbar\omega}{c^2} \int \frac{1}{r^2} \cdot \frac{r^2}{cd} \cos\phi \, d\phi$$

$$= \frac{GM\hbar\omega}{c^3 d} \int_{-\pi/2}^{\pi/2} \cos\phi \, d\phi = \frac{2GM\hbar\omega}{c^3 d}$$

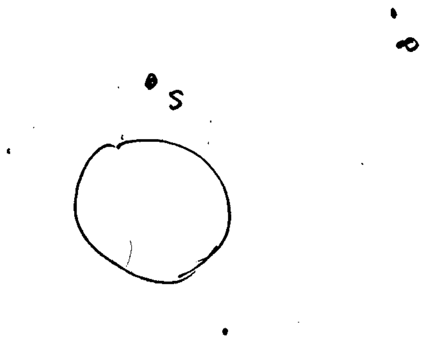
$$\therefore \quad \Delta\theta = \frac{\Delta p}{p} = \frac{\Delta p}{(\hbar\omega)/c} = \frac{2GM}{c^2 d}$$

$$\frac{2GM_\odot}{c^2} = 2.953 \text{ km} \qquad R_\odot = 6.96 \; 10^8 \text{ m}$$

$\therefore$ For a grazing angle

$$\Delta\theta \simeq \frac{2.953 \; 10^3}{6.96 \; 10^8} \simeq 4.2 \; 10^{-6} \text{ radians}$$

$$\simeq 2.4 \; 10^{-4} \text{ degrees.}$$



$$d\tau_s = dt_s \sqrt{1 - v^2/c^2}$$

$$\omega_\infty = \omega_s \left(1 - \frac{GM}{c^2 R}\right)$$

$$dt_\infty = \frac{dt_s}{\left(1 - \frac{GM}{c^2 R}\right)}$$

$\therefore$ Time dilation correction:

$$dt_s = \frac{d\tau_s}{\sqrt{1 - v^2/c^2}} \simeq d\tau_s \left(1 + \tfrac{1}{2} v^2/c^2\right)$$

Gravitational:

$$dt_\infty \simeq dt_s \left(1 + \frac{GM}{c^2 R}\right)$$

For a satellite in Earth orbit:

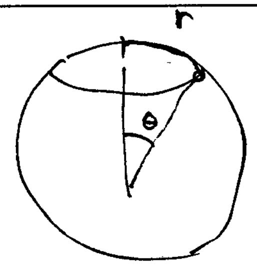
$$\frac{m v^2}{R} = \frac{G m M}{R^2} \Rightarrow v^2 = \frac{GM}{R}$$

$\therefore$ Time dilation $= \left(1 + \tfrac{1}{2}\epsilon\right)$

Gravity $= (1 + \epsilon)$

$$\epsilon = \frac{GM}{c^2 R} = \frac{8.87 \text{ mm}}{2 . (2.7\ 10^4 \text{ km})} = 1.64\ 10^{-10}$$

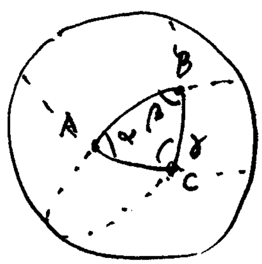
This corresponds to a distance error $\epsilon c t$ over a time duration t = .049 m/s



$$C = 2\pi a \sin\theta$$

$$r = a\theta$$

$$\frac{C}{2\pi r} = \frac{\sin\theta}{\theta} \simeq 1 - \frac{1}{6}\theta^2 \simeq 1 - \frac{1}{6}\left(\frac{r}{a}\right)^2$$



Let ABC = spherical triangle.

$A'B'C'$ = antipodal points to ABC.

O = center of sphere.

planes OAB OAC cut out a wedge (lune)

with ~~area~~ solid angle $\frac{\alpha}{2\pi} \cdot 4\pi = 2\alpha$

$$2(\alpha+\beta+\gamma) = \Omega(\text{lune } OAB, OAC) + \Omega(\overset{\text{lune}}{OAC, OCB}) + \Omega(\overset{\text{lune}}{OCB, OBA})$$

$$\Omega(\overset{\text{lune}}{OAB, OAC}) = \Omega(\text{triangle } ABC) + \Omega(\text{triangle } A'BC)$$

$$\Omega(\text{tringle } A'BC) = \Omega(\text{triangle } AB'C')$$

$$\therefore\ 2(\alpha+\beta+\gamma) = 3\,\Omega(ABC) + \Omega(AB'C') + \Omega(ABC') + \Omega(AB'C)$$

$$\Omega(ABC) + \Omega(AB'C') + \Omega(ABC') + \Omega(AB'C) = \Omega(\tfrac{1}{2}\text{ sphere}) = 2\pi$$

$$\therefore\ 2(\alpha+\beta+\gamma) = 2\,\Omega(ABC) + 2\pi$$