1. Compute the Riemann tensor for the metric
$$ds^2 = e^{2A(r)} dr^2 + r^2 d\Omega^2 - e^{2B(r)} dt^2$$
using the Cartan method, and then compute the Ricci tensor and Ricci scalar.

2. Prove the geodesic deviation equation
$$(\nabla_u \nabla_u \, y)^\mu = -R^\mu{}_{\lambda\alpha\beta} u^\lambda u^\beta y^\alpha$$
where u is the tangent vector of a geodesic. Do this by considering a geodesic $x^\mu(s)$ and a neighboring geodesic $x^\mu(s) + y^\mu(s)$, where y is infinitesimal. Assume torsion vanishes.

3. Show that (no torsion)
$$V^\alpha{}_{;\nu\mu} - V^\alpha{}_{;\mu\nu} = R^\alpha{}_{\beta\mu\nu} V^\beta \tag{1}$$

4. Show that for Riemann normal coordinates at a point $\mathcal{P}_0$

(a)
$$\left(\frac{\partial}{\partial x^\alpha}\right)_{\mathcal{P}_0} = e_\alpha(\mathcal{P}_0)$$

(b)
$$\Gamma^\alpha_{\beta\gamma}(\mathcal{P}_0) = 0$$

(c)
$$\left.\frac{\partial \Gamma^\alpha_{\beta\gamma}}{\partial x^\lambda}\right|_{\mathcal{P}_0} = -\frac{1}{3}\left(R^\alpha{}_{\beta\gamma\lambda} + R^\alpha{}_{\gamma\beta\lambda}\right)_{\mathcal{P}_0}$$

If there is a metric, and $g(e_\alpha, e_\beta) = \eta_{\alpha\beta}$, then one has in addition

(d) $g_{\alpha\beta}(\mathcal{P}_0) = \eta_{\alpha\beta}$

(e) $g_{\alpha\beta,\lambda}(\mathcal{P}_0) = 0$

(f)
$$g_{\alpha\beta,\lambda\tau}(\mathcal{P}_0) = -\frac{1}{3}\left(R_{\beta\lambda\alpha\tau} + R_{\alpha\lambda\beta\tau}\right)_{\mathcal{P}_0}$$

(g)
$$g_{\alpha\beta,\lambda\tau}(\mathcal{P}_0) - g_{\alpha\tau,\lambda\beta}(\mathcal{P}_0) = R_{\alpha\lambda\tau\beta}(\mathcal{P}_0)$$