A.V. Manohar **Ph225A: General Relativity** Problem Set 4

Show that:

1. For any invertible square matrix M
$$\delta \ln \det M = \operatorname{Tr} M^{-1} \delta M$$

2. $$\Gamma^{\mu}_{\mu\lambda} = \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial x^{\lambda}} \sqrt{|g|}$$

3. $$V^{\mu}{}_{;\mu} = \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial x^{\mu}} \left(\sqrt{|g|} V^{\mu} \right)$$

4. $V_{\mu;\nu} - V_{\nu;\mu} = V_{\mu,\nu} - V_{\nu,\mu}$ but that $V^{\mu}{}_{;\nu} - V^{\nu}{}_{;\mu} \neq V^{\mu}{}_{,\nu} - V^{\nu}{}_{,\mu}$. Assume no torsion.

5. For an antisymmetric two-index tensor,
$$F^{\mu\nu}{}_{;\mu} = \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial x^{\mu}} \left(\sqrt{|g|} F^{\mu\nu} \right)$$

6. For the covariant Laplacian ∇^2:
$$\nabla^2 f = \frac{1}{\sqrt{|g|}} \partial_{\mu} \left(\sqrt{|g|} g^{\mu\nu} \partial_{\nu} f \right)$$