1. You are given a vector field (A^x, A^y, A^z) in Cartesian coordinates. Compute (A^r, A^θ, A^ϕ) and (A_r, A_θ, A_ϕ) in spherical polar coordinates in terms of (A^x, A^y, A^z). Also compute the usual r, θ, ϕ components, $\mathbf{A}\cdot\hat{\mathbf{r}}$, $\mathbf{A}\cdot\hat{\boldsymbol{\theta}}$, $\mathbf{A}\cdot\hat{\boldsymbol{\phi}}$.

2. Convert the one-form $\omega = x\,\mathrm{d}y - y\,\mathrm{d}x$ to polar coordinates. Compute $\mathrm{d}\omega$ in both coordinate systems, and verify that the polar coordinate result is the same as converting the Cartesian $\mathrm{d}\omega$.

3. Given a function $f = r\cos\theta$, compute $\mathrm{d}f$ and $\mathrm{d}\mathrm{d}f$.

4. Given a one-form vector potential $A = A_\mu dx^\mu$, compute $F = \mathrm{d}A$ and show that its components are the electric magnetic fields. Compute $\mathrm{d}F$ (without assuming $F = \mathrm{d}A$), and show that $\mathrm{d}F = 0$ are the source-free Maxwell equations.

5. Let $\omega_1 = \sin\theta\,\mathrm{d}\phi$. Verify Stokes' theorem

$$\int_M \mathrm{d}\omega_1 = \int_{\partial M} \omega_1$$

where M is the Northern hemisphere of a unit sphere. Repeat for $\omega_2 = \cos\theta\,\mathrm{d}\phi$. Why does Stokes' theorem fail in this case?