

1. A particle moves along a vertical trajectory in a uniform gravitational field, starting at $h = 0$ at $t = 0$, and returning to $h = 0$ at $t = T$. Find the trajectory which extremizes the time difference measured between a clock on the particle, and a clock at rest on the ground at $h = 0$, in the non-relativistic weak-field limit, i.e. to lowest non-trivial order in v and g .
2. Compute the deflection of light by the Sun, using Newtonian gravity, and treating the photon as a particle with mass $\hbar\omega/c^2$. Note that the deflection angle $\theta \ll 1$.
3. GPS satellites orbit the earth with an orbital radius of 2.7×10^4 km. Find the time dilation and gravitational corrections to a clock on the satellite, as compared with an inertial observer at infinity.
4. Compute the ratio of the circumference C to $2\pi r$ for a circle, and the deficit angle $\sum \theta_i - \pi$ for a triangle, if they are drawn on the surface of a two-sphere of radius a . For a small circle, show that

$$\frac{C}{2\pi r} = 1 - \alpha r^2 + \dots$$

and find α . For the triangle, show that

$$\sum \theta_i - \pi = \Omega$$

where Ω is the solid angle subtended by the triangle at the center of the sphere.